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On the critical exponent of η/s and a new exponent-less measure of fluidity

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Abstract

We discuss on the critical exponent of η/s for a fluid, and propose a new exponent-less measure of fluidity based on a mode-mode coupling theory. This exhibits a remarkable universality for fluids obeying a liquid-gas phase transition both in hydrodynamic as well as in nonhydrodynamic region. We show that this result is independent of the choice of the fluid dynamics, *viz.*, relativistic or nonrelativistic. Quark-Gluon Plasma, being a hot relativistic and a nearly perfect fluid produced in relativistic heavy-ion collisions, is expected to obey the same universality constrained by both the viscous and the thermal flow modes in it. We also show that if the elliptic flow data in RHIC puts a constraint on η/s then the new fluidity measure for Quark-Gluon Plasma in turn also restricts the other transport coefficient, *viz.*, the thermal conductivity.

Key words: Relativistic Fluid, Quark-Gluon Plasma, Heavy-Ion Collision, Viscosity, Thermal Conductivity

Recent results [1, 2] from the Relativistic Heavy Ion Collider (RHIC) at BNL reveal surprising and intriguing dynamical properties of the Quark-Gluon Plasma (QGP). In non-central collisions, the anisotropy with respect to the reaction plane, *i.e.*, the elliptic flow coefficient v_2[2], can well be described up to transverse momenta of order 1.5 GeV/c by the nearly ideal hydrodynamics with small shear viscosity, η. This is much smaller than that obtained in the perturbative quark-gluon plasma [3]. This suggests that the matter produced in the early phase of the RHIC collisions is strongly interacting with nearly perfect fluidity represented by a very small ratio of shear viscosity to entropy density, η/s. The supporting arguments for it have come from various directions: *viz*, viscous hydrodynamics [4, 5] sets an upper bound, $\eta/s \leq (5/4\pi)$; the gauge-gravity dual theory [6] conjectures at universal lower bound, $\eta/s \geq 1/4\pi$; and the Heisenberg's uncertainty principle implies a lower bound [6, 7], $\eta/s \sim 1$ (we use units with $\hbar = k_B = c = 1$). The fluidity of QCD fluid has also been computed from numerical simulation in lattice [8], non-perturbative models [9, 10] and has been compared [12, 11, 13] with commonly known fluids. Moreover, for fluids η is either a

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nondiverging or a very weakly diverging quantity [14, 7] with an exponent ~ -0.04. Thus, the fluidity of a hot viscous QCD matter remains an open as well as interesting problem that requires a much desired exploration of the fluid mechanics.

One can now ask what is the physical quantity that would reflect a good measure of fluidity of a hot viscous fluid or in this context how much relevant is the η/s, which is formed by two quantities, completely different in nature as η is a dynamical transport coefficient whereas s is a thermodynamic one. A possible explanation lies within the general structure of any diffusion constant may it be thermal diffusion, $\mathscr{D}_\kappa$ related to the diffusive decay of longitudinal components of momentum fluctuation or the viscous diffusion, $\mathscr{D}_\eta$ related to the diffusive decay of transverse components of momentum fluctuation of fluids. In this letter we try to address this important aspect of a hot fluid based on fluid dynamics.

Dynamical properties of a many particle system can be investigated by employing an external probe, which disturbs the system only slightly in its equilibrium state, and by measuring the response of the system to this external perturbation. A large number of experiments belong to this category, such as studies of various line shapes, acoustic attenuation, and transport behaviour. In all these experiments, one probes the dynamical behaviour of the spontaneous fluctuations in the equilibrium state. In general, the fluctuations are related to the correlation function, which provide important inputs for quantitative calculations of complicated many-body system.

The density-density correlation function is defined as

$$\sigma_{nn}(\mathbf{r},t) = \langle \delta n(\mathbf{r},t)\delta n(\mathbf{0},0)\rangle \ , \tag{1}$$

where the angular bracket denotes an equilibrium ensemble average, and $\delta n(\mathbf{r},t) = n(\mathbf{r},t) - \langle n \rangle$, is the local deviation of the number density $n(\mathbf{r},t)$ from the equilibrium value of the number density. The spectral function can be obtained by Fourier transformation of (1) as

$$\rho_{nn}(\mathbf{q},\omega) = \int d^3\mathbf{r} \int_{-\infty}^{\infty} dt \ \sigma_{nn}(\mathbf{r},t) \ e^{-i(\mathbf{q}.\mathbf{r}-\omega t)} \ , \tag{2}$$

and the static correlation function can be defined as

$$\tau_{nn}(\mathbf{q}) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \rho_{nn}(\mathbf{q},\omega) \ . \tag{3}$$

Now, within hydrodynamical description the correlation function can be calculated only when wave number of the dynamical density fluctuation, q is much smaller that the inverse correlation length, ξ ($q\xi << 1$), or, equivalently wavelength, λ of density fluctuation is appreciably larger than the correlation length ($\lambda >> \xi$). In the case of nonrelativistic fluids, the dynamical density fluctuation based on Navier-Stokes equation [15] in the hydrodynamic limit as well in the nonhydrodynamic limit (*viz.*, around critical point) has been studied [16] for Newtonian fluids in details. Recently, a relativistic generalisation of density fluctuation is obtained [17] using dissipative relativistic fluid dynamics coupled with conserved quantities and the dynamical structure

function is

$$\rho_{nn}(\mathbf{q},\omega)/\tau_{nn}(\mathbf{q}) = \left(1-\frac{1}{\gamma}\right)\frac{2\mathscr{D}_\kappa q^2}{\omega^2+(\mathscr{D}_\kappa q^2)^2}+\frac{1}{\gamma} \left\{\frac{\mathscr{D}_s q^2/2}{(\omega-c_s q)^2+(\frac{\mathscr{D}_s q^2}{2})^2} + \frac{\mathscr{D}_s q^2/2}{(\omega+c_s q)^2+(\frac{\mathscr{D}_s q^2}{2})^2}\right\}, \tag{4}$$

where γ is the ratio of specific heats, $\tilde{C}_P = T(\partial S/\partial T)_P$ at constant pressure to $\tilde{C}_n = T(\partial S/\partial T)_n$ at constant density with S is the equilibrium entropy per particle, c_s is the speed of sound propagation with the damping constant $\mathscr{D}_s$ and $\mathscr{D}_\kappa$ is the thermal diffusivity. The higher-order terms involving the quantities $\mathscr{D}_\kappa q/c_s$ and $\mathscr{D}_s q/c_s$ are neglected [1]. The thermal diffusivity is defined as,

$$\mathscr{D}_\kappa = \frac{\kappa}{n_0\tilde{C}_P} \ , \tag{5}$$

where κ is the thermal conductivity and n_0 is the equilibrium number density of fluid. Note that the nonrelativistic equivalence of $n_0\tilde{C}_P$ in (5) is $\rho_0 C_p$, where ρ_0 is the equilibrium mass density.

The sound wave damping constant is given as

$$\mathscr{D}_s = \mathscr{D}_\kappa(\gamma-1)+\frac{\frac{4}{3}\eta+\zeta}{w_0}+c_s^2T^2\left(\frac{\kappa}{w_0}-2\mathscr{D}_\kappa\alpha_P\right) \ , \tag{6}$$

where η and ζ are, respectively, shear and bulk viscosities, w_0 is the equilibrium enthalpy density and α_P is the thermal expansivity at constant pressure. The third term in (6) is purely relativistic correction whereas the second term is the minimal relativistic one due to the appearance of w_0 instead of ρ_0.

The dynamical structure function of density fluctuation for relativistic dissipative fluids in (4) is a sum of three Lorentzian line shapes with general form $f(\omega) = 2\Gamma/[\Gamma^2+(\omega-\omega')^2]$, centered at the frequency ω' with a half-width at half maximum given by Γ. The Rayleigh component, is centered about $\omega = 0$, with a half width

$$\Gamma_R = \mathscr{D}_\kappa q^2 \ . \tag{7}$$

The Brillouin doublets, are located symmetrically about $\omega = 0$ at the frequencies $\omega_B^\pm = \pm c_s q$, each one with a half-width given by

$$\Gamma_B = \frac{1}{2}\mathscr{D}_s q^2 \ . \tag{8}$$

[1] A relativistic first order equation, such as that by Landau or by Eckart, is parabolic and formally violates the causality, and hence acausal. The causality problem can be circumvented with the inclusion of the second-order in derivative expansion [4, 17, 18]. However, this does not affect the spectral function of dynamical density fluctuation.

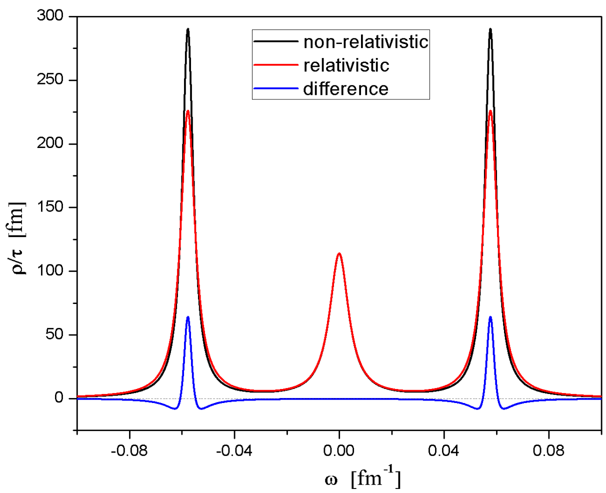

Figure 1: (Colour online) The structure function ρ_{nn}/τ_{nn} as a function of ω in the minimal relativistic fluid (red curve), nonrelativistic fluid (black curve) and their difference (blue curve) for parameters [17] $q = 0.1$ fm^{-1}, chemical potential $\mu = 200$ MeV, $T = 200$ MeV, $\eta/(n_0S) = \zeta/(n_0S) = 0.3$ and $\kappa T/(n_0S) = 0.6$. The minimal relativistic correction shows up in the Brillouin doublets but not in the Rayleigh component centered at $\omega = 0$ (see text).

We note that in (4) the transverse component of the momentum fluctuation decouples from the density fluctuation but admits a diffusive solution that relaxes exponentially with the viscous diffusion, $\mathscr{D}_\eta$ as

$$\mathscr{D}_\eta = \frac{\eta}{w_0} \ , \tag{9}$$

which reduces to η/ρ_0 in the nonrelativistic limit. The knowledge of the line widths in (7) and (8) along with the three diffusion equations (5), (6) and (9) are sufficient to determine the three transport coefficients, κ, η and ζ.

Now we note that the peak and the width of the Brillouin doublets obviously change due to the relativistic corrections of the sound mode, $\mathscr{D}_s$ in (6). In contrast the Rayleigh peak as well as the width Γ_R remain same for both nonrelativistic and relativistic fluids because the thermal diffusion $\mathscr{D}_\kappa$ does not change due to the equivalence between $n_0\tilde{C}_P$ and $\rho_0 C_P$. We demonstrate this aspect in Fig. 1 with only the minimal relativistic corrections in (6). Further, the line width Γ_R is controlled by the dominant behaviour of either κ or $\tilde{C}_P$ in $\mathscr{D}_\kappa$. The behaviour of $\mathscr{D}_s$ will control that of Γ_B through the relative behaviours of $\kappa/\tilde{C}_n$, η and ζ. Also the strength of the various peaks in (4) depends on γ. This hydrodynamics predictions have been used in the limit $T \to T_C$ to study the behaviour of relativistic fluid [17] around critical point. The fundamental assumption of hydrodynamic, $q\xi << 1$, close to T_C is no longer valid but it was argued that for a given q there will be always a temperature range over which hydrodynamics predictions should be reliable. Around the critical point γ diverges faster than the viscosities (*viz.*,

ζ), one can easily find from (4) that the sound mode gets attenuated and the only surviving soft mode is the diffusive thermal mode. This has important consequence as we will see below.

Generally, the long wavelength part of the density fluctuation around critical point is very intense that could induce a velocity gradient in the boundary of the fluid, which would dissipate energy. This dissipation can be interpreted in terms of the mode-mode coupling [19, 20]. For instance, as discussed above, the heat flow mode (thermal diffusion) decays into shear viscous mode plus a thermal mode. One can obtain this decay mode in terms of the relevant transport coefficients.

Information on critical behaviour of fluids are generally extracted from scattering experiments performed on the fluids [21]. To analyse these data Swinney and Henry [22], based on mode-mode-coupling theory, obtained a particular dimensionless combination of measured quantities for wide range of fluids obeying liquid-gas phase transition as

$$\Gamma^* = \frac{6\pi\eta\Gamma_R}{Tq^3}. \tag{10}$$

Using the measured values of Γ_R, η and the correlation length, ξ for different temperatures and scattering angles ($q = 4\pi\sin(\theta/2)/\lambda$, θ is the angle of scattering), obtained for various sample of multiple component fluids [21], when plotted Γ^* as a function of $q\xi$ all fluids obeying liquid-gas phase transition fall on a single universal curve as shown in Fig. 2. The single curve describes the critical behaviour not only along the critical isochore and the coexistence curve, but also along any thermodynamic path in the critical region. It is clear from Fig. 2 that,

$$\Gamma^* = \begin{cases} \frac{1}{q\xi} & \text{for } q\xi < 1 \text{ (hydrodynamic domain)}, \\ 1 & \text{for } q\xi \geq 1 \text{ (near critical point)}, \end{cases} \tag{11}$$

provided $\frac{6\pi\eta\Gamma_R\xi}{Tq^2} = 1$, and using (7) it becomes

$$\mathscr{F}_{\eta\kappa} = \frac{\eta}{(\mathscr{D}_\kappa\xi)^{-1}\,T} = \frac{1}{6\pi}, \tag{12}$$

which is independent of the nature of the fluid despite the various microscopic details. This is also consistent with Stokes formula for the mobility of a sphere of radius ξ moving through a viscous liquid with a nondiverging shear viscosity, η.

Now, the equality, $\Gamma^* = 1$ in (11) basically holds at and near the critical point, indicating the length scale matching around the critical point. This implies that instead of diverging without any upper bound, the correlation length of fluid becomes of the order of the wavelength of the density fluctuation ($\xi \sim \lambda$). On the other hand this also suggests that $\mathscr{D}_\kappa\xi$ in together is a nondiverging quantity around critical point since both of them are expected to have same critical exponent [15] but opposite in sign. The dimensionless quantity, $\mathscr{F}_{\eta\kappa}$ associated with the viscous and the thermal modes together, could then be regarded as a good fluidity measure of a hot viscous fluid obeying liquid-gas phase transition, which remains at a universal value $1/(6\pi)$ for both nonhydrodynamical as well as in hydrodynamical region.

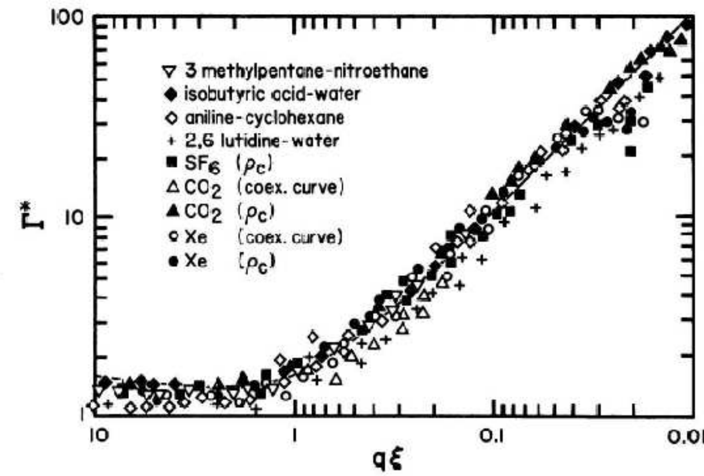

Figure 2: Γ^* as a function of $q\xi$ for several several substances that cover a wide range of molar mass, chemical structure and complexity and it is adopted from [22]. Note that the measured data are from various experiments and the detailed references of those can also be found in [21, 22, 23].

We now note that the important quantity that enters in (11) and (12) is the Rayleigh width, Γ_R. The analysis, based on the Navier-Stokes theory presented in the beginning of this letter, clearly exhibits that the relativistic correction does not show up in the Rayleigh width Γ_R in (7). It indicates that the fluidity, $\mathscr{F}_{\eta\kappa}$ is, obviously, independent of the choice of the (*viz.*, relativistic or nonrelativistic) fluid dynamics. *This possibly indicates that a good fluid, either relativistic or nonrelativistic, is always a good fluid irrespective of the other details of the microscopic degrees of freedom.*

In a heavy-ion collisions, the produced matter has a strong flow initially along the beam axis. If the matter is viscous the viscosity then counteracts this by reducing the effective longitudinal pressure and thus enhancing it in the transverse plane. Therefore, the viscosity leads to a larger radial flow than that of the ideal fluid and also to an angular modulated radial flow (v_2) due to the velocity gradient. This is indeed the situation in the experiments at RHIC [1, 2] that probably probed a QGP at a temperature little above T_C with a small value of η/s [4, 5, 12, 11] implying that the QCD matter produced in such experiments is a nearly perfect fluid and likely to obey a liquid-gas phase transition with a critical end point [24]. The spectral analysis for relativistic fluid in (4) or in Fig. 1 represents that a small density perturbation propagates at the speed of sound and is eventually damped near the critical point. The only surviving mode is the diffusive heat flow mode. The observed fluidity could be interpreted on the basis of mode-mode coupling theory as the decay of a longitudinal heat flow mode in QGP to a viscous and a thermal mode in which the transverse viscous mode reduces the thermal flow in the QGP. The QGP fluid produced in RHIC belongs to the same universality class defined by the fluidity measure in (12) that interrelates the relevant

transport coefficients, *viz.*, the shear viscosity η, and the thermal conductivity κ.

We now recall that for fluids η is either be nondiverging or weakly diverging (with an exponent ~ -0.04) [7, 14]. One can expect that η/s should also be a nondiverging or weakly diverging quantity as entropy generally does not show any critical exponent. Surprisingly, for simple fluids as well as for QGP η/s (as obtained from lattice QCD simulation [8] and also from the viscous hydrodynamic calculations [4, 5, 12] in view of recent RHIC data [2]) shows an exponent (see Fig.3 of Ref. [12]) of the order of 'one'. This observed feature of η/s can clearly be seen from (12) as

$$\frac{\eta}{s} = \mathscr{F}_{\eta\kappa}\frac{\mu}{S}(\kappa\xi)^{-1}, \tag{13}$$

where inverse of $\kappa\xi$ in together has an exponent ~ 1.2 for liquid-gas phase transition [15].

For various fluids as discussed in Refs. [12, 23] one can parametrise η/s with the reduced temperature, $\varepsilon = |T/T_C - 1|$, in the domain $T_C \leq T \leq 2T_C$ as

$$\frac{\eta}{s} \sim (a\varepsilon^{\delta} + b)\ , \tag{14}$$

where a and b are different for different fluids. For QGP [8, 12] the elliptic flow data in RHIC [2] restrict those as $\delta \sim 1$, $a \sim 0.64 \sim 2/3$ and $b \sim 0.1 \sim 1/(4\pi)$, which describe the viscous mode despite the various microscopic features in it. Obviously, the new fluidity measure $\mathscr{F}_{\eta\kappa}$ in (12) then puts a constraint on the heat flow mode in QGP, which in turn restricts the associated transport coefficient κ as

$$\kappa = \mathscr{F}_{\eta\kappa}(a\varepsilon^{\delta} + b)^{-1}\frac{\mu}{S\xi}\ , \tag{15}$$

where the correlation length, ξ is related to various moments of the net baryon number fluctuations [25].

In view of nearly perfect fluidity nature of QGP observed in RHIC we suggest through a new exponent-free fluidity that the thermal conductivity, which is a measure of transport of energy by the particles, also becomes an important quantity to look at. This has not been discussed before in the literature in this context. Interestingly, it has been proposed to measure the thermal conductivity in RHIC with its upgradation [26], which could then test the above estimation of this quantity both in hydrodynamical and nonhydrodynamical regions, and in turn our proposal of new fluidity measure. It will also be very interesting, if one can find a possible way to check this universal behaviour of fluidity through lattice QCD calculations and in nonperturbative models. The Large Hadron Collider (LHC) at CERN will produce a QGP at much higher temperature (at least twice of RHIC) and μ close to zero, which would then cool and pass through the critical region and it would be interesting to see whether there could still be a Rayleigh peak in this limit.

We note that our analysis is based on the mode-mode coupling theory and the relativistic Navier-Stokes theory. A basic assumption of this mode-mode-coupling formulation is the nondivergence of shear viscosity whereas a proper RG calculation reveals that shear viscosity has a very weak divergence governed by the exponent -0.04. On

the other hand a Navier-Stokes theory is a kind of mean field equation and may not be very appropriate to discuss the dynamics around critical point. The dynamics of the fluid in and around the critical point should, however, be described by the evolution equation derived from a microscopic theory. Nonetheless, the present understanding of the critical dynamics of the produced QCD matter in RHIC experiments is meagre and one needs to depend on a kind of mean field description to qualitatively understand the behaviour of the system in and around the critical point because the hydrodynamic variables become comparable to the time scale of an order parameter around the critical point due to critical slowing down.

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